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CRITICISMS AND DISCUSSIONS.

THE ECONOMY OF THOUGHT.

I.

From the earliest times until the seventeenth century mathematicians were chiefly occupied with particular questions—the properties of particular numbers and the geometrical properties of particular figures, together with simple mechanical questions concerning centers of gravity, the lever and so on. The only exception to this was afforded by algebra, in which symbols (like our present x and y) took the place of numbers, so that, what is a great advance in economy of thought and other labor, a part of calculation could be done with symbols instead of numbers, so that the *one* result stated a proposition valid for a whole class (often an infinity) of different numbers. Such a result is that which we now write:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,$$

which remains valid when we substitute any particular numbers for a and b, and labor in calculation is often saved by the formula even in this very simple case.

This reflection, which is quite an evident one, came in an article of mine on "The Relevance of Mathematics" published in 1909.[1] In a note, I referred to some criticisms on Mach's idea of the economy of thought in science, which had been made by Mr. N. R. Campbell.[2] Mr. Campbell's objections seemed and still seem to me to rest on a misunderstanding of what Mach really meant. This is part of the note:

"Perhaps the phrase 'economy of thought' is not well chosen, and may lead to such misunderstandings; the principle is meant to direct attention to a rule of scientific method which can be readily admitted, and certainly the goal of science will not 'have been attained when its students, guided by this principle, have ceased to think.'

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[1] *Nature,* Vol. LXXX, 1909, pp. 382-384.

[2] *The New Quarterly,* Vol. I, 1908, p. 498.

This rule may be thus described. As science advances, besides actually overcoming an obstacle, it consciously or unconsciously leaves marks of guidance for those who come after; so that those obstacles which required great genius to overcome in the first instance afterwards became quite easily so. This is necessary in order that our energies may not be spent by the time that we reach a new obstacle not hitherto surmounted; and 'economy of thought' means the sparing of waste of the energy of thought whilst treading the path already trodden by our predecessors, so that we may keep thought for the really important new problems—not that we may cease to think about new problems.

"And thus we have legacies left by great men, such as Lagrange's analytical mechanics and Fourier's theory of the conduction of heat, which are merely inventories of extensive classes of facts, arranged with wonderful compactness.[3] In this description of an infinity, perhaps, of facts by a *few* formulas, there is undoubtedly an esthetic motive and value; but apart from this there is this important economical aspect: that a multitude of particular facts and 'laws,' which we had hitherto to remember, actually or artificially (in a note-book or library), is, in the theory, comprised in a few symbolical formulas, which only require *logical* development to get at the particular cases. From this point of view we get the apparent paradox that 'economy of thought' leads to the replacement of memory by reason. The solution of the paradox is that logical development can be made more mechanical even than memory, and that thus thought is spared, so that we can concentrate it on the unsolved problems which are always coming into our field of vision as we advance.

"The tendency to economy of thought, which is shown in the growth of physics—for example, in the inclusion of the particular Biot's law of the distribution of temperatures in Fourier's theory[4]—may also be seen in the symbolism of pure mathematics."

II.

This criticism of a criticism led to a hitherto unpublished discussion between Mr. Campbell and myself. As I think there are points of interest about this discussion, I proceed to give some account of it.

Mr. Campbell replied to my remarks on his criticisms of the principle of the economy of thought in a letter of May 28, 1909, addressed to the editor of *Nature.* This letter and the subsequent discussion were too long for the correspondence columns of *Nature,* and through the courtesy of Mr. Campbell I am here enabled to give a reproduction of his letter:

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[3] Cf. Mach, *Die Principien der Wärmelehre,* 2d ed., Leipsic, 1900. p. 115.

[4] Cf. *ibid.,* p. 81.

"In his article on 'The Relevance of Mathematics' Mr. Jourdain seeks to refute some criticisms of Professor Mach's principle of 'economy of thought,' which I made in a recent number of the *New Quarterly Review.* Since my reference to Professor Mach was only incidental, perhaps I may be allowed space to elaborate my position a little further in reply to Mr. Jourdain.

"Mr. Jourdain says that Professor Mach does not mean what he says, and in this I agree with him. If I imagined that he really intended to assert so ridiculous a principle, I should not have so very great a respect for his writings. But undoubtedly he has been taken literally by many people, and it is therefore permissible and desirable to criticize the letter as well as the spirit of his statements. Accordingly I must point out again that, if the object of science is the economy of thought, the best science is that which requires no thought: 'The object of science will be attained when its students have ceased to think.' Nobody can consistently maintain the literal interpretation of Professor Mach's principle and hold at the same time that science has the smallest intellectual value.

"According to Mr. Jourdain the principle that Professor Mach intends to assert is that of the economy of memory at the expense of reason. I repeat that this may be what Professor Mach means, but it is certainly not what he says; for reason is as much 'thought' as memory. But, if the principle is put in this form, I agree readily that it describes roughly the motive of a great part of what is ordinarily termed 'science,' for memory is the most obvious example of a form of thought which has no esthetic value for most minds. Laws are, indeed, partly efforts at the economy of memory at the expense of reason, but their value lies at least as much in the expenditure of reason as in the economy of memory. Landolt and Börnstein's tables are far more efficient in saving memory than all the laws that were ever discovered, but nobody (except, perhaps, certain chemists) would consider them the ideal of a science.

"It is in connection with theories that the principle of the 'economy of thought,' taken literally or interpreted by Mr. Jourdain, is so utterly absurd. (I have explained the distinction between a law and a theory in the article to which Mr. Jourdain refers.) It is perfectly true that theories *might* save memory, but it is perfectly untrue that they *do* save it, or that it is for their economizing properties that they are valued by men of science. For conviction on this point Mr. Jourdain has only to examine our practice. Does a physicist, when he wants to know the effect upon the volume of a given mass of gas of changing the pressure, reflect for a moment upon the kinetic theory of gases? Would an astronomer, accosted suddenly, confess complete ignorance of Kepler's laws and ask for time to examine the consequences of the theory of gravitation? Of course not. No man in his senses ever reconciled himself to forgetting a law because he knew of a theory by which it was explained, for the simple reason that the mental effort of deducing the law from the theory is even less pleasurable than that of remembering the law. A theory economizes nothing. It is a pure addition to our thought, and it is valuable in proportion to the value of that which it adds. If science were merely an elaborate method of pampering the mental indolence of mankind, it would be better left to those who care for nothing but a base utilitarianism. It is because theories provide us with ideas the contemplation of which is of the very highest esthetic value, that science is a worthy employment for the noblest intellects.

"The failure to distinguish clearly between the relatively unimportant law and the essential theory, which marks the work of most of those who write of the 'philosophy of science,' is due largely, I think, to the fact that so many of these authors are primarily mathematicians. They naturally seek for some motive in science which presents an analogy to that which inspires their own splendid study. The rationale of pure mathematics is, if I understand rightly the treatise of Mr. Bertrand Russell, absolutely unique among all forms of thought, but there is in its methods some element which shows a rough, a very rough, analogy to the formation of scientific laws. But no opinion could be more erroneous than the prevalent view that mathematics and science are closely connected species of a single genus of thought. The essence of a science is its theories, and for a theory the analogy is to be found not in mathematics but in metaphysics, and still more in art. The debt of physicists to mathematicians is incalculable, but when their long and fruitful intimacy leads to the development of a '*Naturphilosophie*' which respects nothing but 'facts,' and to the glorification of that magnificent lie, *Hypotheses non fingo,* on that part of him who alone in his stupendous genius could embrace the highest developments of instincts so diverse as the mathematical and the physical, it is time to insist that it is a mere friendship and not a deep tie of blood and lineage."

III.

This letter, which was forwarded to me at the time (June 3, 1909) by the editor of *Nature,* suggested to me some reflections of which the following is a copy:

"The chief difference between Mr. Campbell and myself seems to be concerning the ends of science. With Mach, I believe that the object of natural science is to complete in thought what we observe only slowly and partially; this object seems to me, at bottom, a practical one. It is true that science gives us noble ideas for our contemplation; this fact comes home to one in spite of its constant repetition in works on popular science. But it is, I think, a mistake to lay exclusive stress, as Mr. Campbell seems to do, on this purely intellectual function of science, and to depreciate its more obviously utilitarian aspects. Even if astronomy had no other purpose than to produce the *Nautical Almanac* it would be hard on it to call its purpose "base." If Mr. Campbell is right, then: (1) the essence of science is much more like art than I had supposed; (2) not only have many readers misunderstood Mr. Russell's work, but also Mr. Russell himself, for he maintains that the rationale[5] of pure mathematics is not unique among forms of thought, but *is* logic; and lastly (3) science is merely friendly to logic and not otherwise related to it. So Mr. Campbell's rightness would mean that the hopes of many of us must be given up.

"Until I read Mr. Campbell's letter, I maintained that the real meaning of Mach's principle of the economy of thought in science was that of the replacement of memory by reason. In fact, the obvious saving of labor occasioned by a formula valid for an infinity of cases, as compared with a set of tables which, often inaccurately and always incompletely,[6] try to fulfil the same purpose, appeared to me so supremely important that I overlooked the importance of a literal economy of thought in science, and Mr. Campbell, by attacking what I did not say, but ought to have said, has rendered me a great service.

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[5] [Here I take this word to mean the *essence* and not the *methods* of mathematics. The methods used to discover logical propositions need not, of course, be logical.]

[6] "By this, I mean, of course, that no table of the values of a function can give the infinity of particular values of it that we can obtain from a formula. If this is said to be immaterial for practical purposes, there is a vague hypothesis made in that statement about the nature of the function in question; and it is mathematics alone which can state exactly what this hypothesis is."

"When a man pleads for the economy of thought in physics, there is no more implication that his ideal is a cessation of the need for students to think at all than there is a like implication in his pleading for a symbolic logic. We know how some old-fashioned logicians used to regard symbolic logic: their ignorance of it and its objects merely increased their contempt for it. And by way of reply, I need only refer here to Mr. W. E. Johnson's[7] just emphasis on the fact that 'a symbolic calculus is an instrument for economizing the exertion of intelligence,' which is necessary because the 'intellectual intuition which perceives the truth of laws in their simple—but absolutely universal—form is incompetent to perceive the same truths in more complicated forms.' By a purely mechanical process we do what unaided intelligence cannot do; we do not attempt the impossible,—the total banishment of intelligence from a science concerned with inference.

"But, though we do not wish to banish thought, we do act in such a way as to spare ourselves unnecessary use of it. I am guilty of overlooking this fact, owing to a mathematical bias in favor of a formula over tables. Of course, when one comes to think of it, to abstain from the use of a table of logarithms, and to work out one's logarithm from first principles, would be eccentric. What is most important in the advance from a tabulated function to the law of the function seems to me to be that, by establishing a logical connection between its descriptions of various phenomena it reduces the number of our scientific principles. Before universal gravitation was formulated, we had at least three laws (those of Kepler) of planetary motion. Kepler's laws are still, doubtless, remembered by astronomers—they are not by all mathematicians—, just as tables are used, to economize mental effort that does not add to our knowledge. And Mr. Campbell admits this. We remember Kepler's laws, he says, because 'the mental effort of deducing the law from the theory[8] is even less pleasurable than that of remembering the law.' And yet, to read the first part of Mr. Campbell's letter, one would think that he objected to this admission of his.

"To sum up, Mach's principle, even when taken literally, is not 'ridiculous.' Men of natural science use it to an extent mathematical people hardly realize—even Kepler's laws are used to save trouble. Secondly, science proper is not concerned with its 'intellectual value.' That it has such a value all admit, but even if it had not, the fact would be irrelevant to science; it would merely fail wholly to displace classics where knowledge is not reckoned by its cash value in ordinary affairs of life. Thirdly, even when I admit that what Mr. Campbell calls a 'theory' has a significance which is not only economical but also logical, I still find myself in disagreement with Mr. Campbell, since he insists that there is no relationship between physics and mathematics (and consequently logic). Fourthly, from our wishing to economize thought as much as possible, it does not follow that our ideal is to abolish it."

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[7] "*Mind,* N. S., I, (1892), pp. 3, 5; cf. Mr. A. N. Whitehead's *Treatise on Universal Algebra,* Cambridge, 1898, p. 4.

[8] "What Mr. Campbell here calls the 'theory,' I call the 'formula' (of gravitation)."

IV.

To this discussion I will append a few remarks.

The question is largely one of terminology. Following Mach, I call a "theory" a collection of propositions which describe a domain of experience closely and concisely, and keep the name "hypothesis" for a proposition from which some or (preferably, of course) all of the known facts can be deduced, but which cannot be logically deduced from propositions describing the facts alone. It has occasionally happened that facts hitherto unknown can be deduced from the hypothesis, and certain persons have concluded too readily that such a deduction confirms the hypothesis. However, the hypothesis would only be confirmed if it could be proved that *all* the facts—known and unknown—are in the domain of experience which the hypothesis is supposed to cover. A hypothesis may be true or false; a theory, being a description of the facts alone, is true if, indeed, it does represent the facts.

Mr. Campbell seems to use the word "theory" in the sense in which I have used the word "hypothesis." It may be that "theory" is a more appropriate term for what I have called "hypothesis"; the verbal point need not be argued. And I propose to call Mr. Campbell's "theory" by the name "theory (C)." What is called "the kinetic theory of gases" is a theory (C), and consequently is not what I called a theory. I thought that Mr. Campbell might object to what he called a "theory" being called a "hypothesis," but I am not sure; at least he says that a physicist lies if he says *Hypotheses non fingo.* Further Mr. Campbell used one word "theory" for what I would call the kinetic "hypothesis" of gases and the "law" or "formula" of gravitation.

Mr. Campbell considers that it would be mad to undergo the mental effort of deducing a law from what he calls a "theory," when remembering the law costs less effort; and yet he seems to consider any economy of thought a theory might have as of very small account as compared with the esthetic value of the theory. So far as I can see, it might be more beautiful always to forget laws and deduce them from a theory; then Mr. Campbell would have to admit that the noblest intellects might belong to mad people.

One of the principal functions of science is the reducing of the processes of thought which have already, perhaps with great labor, been gone through by the pioneers of thought, to a mechanism which can be worked by almost anybody without much new thought being required. The point of this is that the energies of those who succeed the pioneers and the systematizers, and the pioneers' work may be spared for the attacking of new problems. Science is not and never will be completed; consequently there will always be new problems, and the state in which all thought is replaced by mere mechanical operations, whose existence would imply that finality in science were reached, will never come to pass. It is this function of science that is the economy of thought; and I emphasized this function in my article more than in my reply. Since my article was written, I have been much encouraged to continue to hold my opinions by reading in Dr. A. N. Whitehead's admirable little book entitled *An Introduction to Mathematics,* the following sentences: "The interesting point to notice is the admirable illustration which this numeral system affords of the enormous importance of a good notation. By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and in effect increases the mental power of the race."[9] Then again:[10] "....by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. It is a profoundly erroneous truism, repeated by all copy-books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them. Operations of thought are like cavalry charges in a battle—they are strictly limited in number, they require fresh horses, and must only be made at decisive moments."

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[9] Home University Library, London and New York, 1911, p. 59.

[10] *Ibid.,* p. 61.

V.

In a little book entitled *The Principles of Electricity,* which has been lately written by Mr. Campbell for the inexpensive and useful series called "The People's Books,"[11] he has stated his views on the nature and function of scientific "theories" very clearly.

A scientific "law" is[12] "a collection of simple statements about the occurrence of sensations, but it is not a mere collection; the statements are arranged in a definite order, and connections between them are asserted. Thus the statement of 'amber has been rubbed' represents part of the collection, and the statement that 'amber attracted light bodies' another part; the whole statement does not merely affirm both of these partial statements, but asserts also a connection between them, in that the first sensations represented by the first partial statement occurred before those represented by the other; the kind of connection between the ultimate statements, in which one set of sensations is affirmed to occur before another, is the one which has attracted far the largest share of the attention of writers on the philosophy of science, and the term 'law' is usually confined to propositions in which this connection is very obvious. But it is by no means the only kind of connection which is asserted in scientific statements. Thus the statement that amber is rubbed implies the statement that there is such a thing as amber, and this statement in its turn, as we have seen, means that there is a thing which is yellow, hard, brittle, and found by the sea. This statement may be again resolved into partial statements, such as there is a thing which is hard, that there is a thing which is brittle, and so on; each of which is a collection of statements of the occurrence of sensations. But here the connection between the partial statements affirmed by the complete statement, is not that one series of sensations occurs after the other; a substance is called amber if it is first observed to be hard and then observed to be brittle, or if the order of these observations is reversed. If, however, we proceed yet further and analyze the statement that this is brittle, we find that it means that after it is hit with a hammer it breaks into pieces; the connection of statements concerning the occurrence of sensations by dividing them into two groups, of which one is subsequent to the other, reappears."

[11] London and Edinburgh: T. C. and E. C. Jack.

[12] *Op. cit.,* pp. 12-13.

We may notice that, according to Mr. Campbell, it would appear that there was no place in natural science for hypothetical propositions. No proposition in natural science is analogous to that expressed by: "Trespassers will be prosecuted," unless there are in fact some trespassers. It is, then, rather misleading to state propositions of the form "electricity does such and such a thing" when no proposition that there is such a thing as electricity is yet known.

"It is important," says Mr. Campbell,[13] "to notice that the connection of invariable sequence is by no means the only one possible, and that it is very seldom, if ever, the only one occurring even in those statements which are universally recognized as laws. In these circumstances, the attempt to confine the term 'law' to those propositions in which this connection is especially obvious, appears to me artificial and misleading; and in the sequel that term will be applied indifferently to any proposition asserting any kind of general connection between the occurrence of sensations. We shall term 'laws' not only such propositions as that, when amber is rubbed, it attracts light bodies, but also such statements as that there is such a substance as amber, which is at the same time yellow, brittle, and the rest."

"And now," continues Mr. Campbell,[14] "we may ask how far will this process of establishing laws carry us; we can ultimately attain one law which will sum up *all* the observations which we have made and from which all those observations can be deduced, or shall we reach a stage when several laws are needed to describe all the observations and find that no further progress in the simplification of the description can be made? To answer this question thoroughly would need much more space and much more searching inquiry than is suitable in this place, but I think that there can be no doubt that the complete signification, resulting in the description of all the observations by one single statement, cannot be effected if that statement is to be a law."

If we remember Fourier's theory of the conduction of heat, we shall, I think, dissent from the conclusion that it is always impossible to find a law which will sum up all the observations made in a particular domain of science, and from which all these experiments result. What we call, after Fourier, the "theory" of the conduction of heat is simply a series of logical developments, made with the greatest mathematical skill and originality, of one proposition based on experiment—Newton's law of cooling. However, Mr. Campbell gives no grounds for his assertion that the connection of all the laws into one statement which shall be itself a law is not possible.

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[13] *Ibid.*, pp. 13-14.

[14] *Ibid.*, p. 17.

Yet these laws can be connected into one statement which is not a law, but another kind of proposition. And then Mr. Campbell[15] goes on to make a very curious justification for the search for this kind of proposition. The student of pure science "is forced to a new development of science beyond the formulation of laws, because the laws, even when he has got them, do not give him the intellectual satisfaction he seeks; he cannot accept them willingly as the end of his labors. It is quite impossible to say why he is not content with laws, just as it is impossible in the last resort to give any reason for an artistic preference, and fortunately there is no need to make the attempt. For my reader is supposed to be the plain man, and nobody feels more strongly than he the unsatisfying character of laws as an ultimate result of science; I can appeal to his own experience. If this little treatise were brought to a conclusion now, and the reader offered only the laws of electrostatics as the whole pronouncement of science on a great field of investigation, I think he would feel not only that the results were extraordinarily meagre, but that they were of the wrong kind. His natural instinct, unless it had been perverted by the mistaken admonitions of some people who ought to know better, would make him inquire 'why'? 'These laws are all very well,' he would say, 'but I have been expecting to be told why, for instance, charged bodies are able to attract uncharged, or only bodies which are conductors can be charged by induction.' That request voices just the need which leads men of science to their greatest discoveries." It is curious so to appeal to the instinct of the "plain man" when, later on,[16] that very sort of man is rather disparaged: "Those results [of science] are attained by flights of imagination of which the plain man is quite incapable."

"The fresh step, then," says Mr. Campbell,[17] "which we are going to take consists in the substitution for the laws which we have discovered of some other proposition or propositions which shall not be laws. And these new propositions have to fulfil two purposes: first, they have to be such that the laws can be deduced from them, and such that they sum up the laws as the laws sum up the individual observations; second, they have to be such that they give the intellectual satisfaction which cannot be obtained from the laws; for this latter purpose they will have to contain ideas which are more familiar than those of the laws." Such propositions are what Mr. Campbell[18] calls "theories," and he says that "it should be clear that it [the 'theory'] does represent a new development, and that it is perfectly possible to state the laws of the phenomena without any reference whatsoever to it. The help which it affords is doubtless due to the fact that it reduces the quite unfamiliar actions observed with the charged glass and silk to the quite familiar action of the transference of a substance from one body to another."

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[15] *Ibid.*, pp 18-19.

[16] *Ibid.*, p. 28.

[17] *Ibid.*, p. 20.

Mr. Campbell[19] is careful to point out the risk that there is in using "theories": "A theory suggests a great deal more than it actually asserts." He emphasizes[20] that "science in its highest form is not opposed to art, it is a form of art"; and he is scornful[21] about the study of science for utilitarian reasons. Of course this scorn is right; but it seems rather ungrateful not to remember that science was first developed from utilitarian reasons.

PHILIP E. B. JOURDAIN.

CAMBRIDGE, ENGLAND.

AN ANSWER TO MR. BERTRAND RUSSELL'S ARTICLE ON THE PHILOSOPHY OF BERGSON.[1]

Mr. Russell concludes his criticisms of Bergson's philosophy with the accusation that he confuses subject and object in his theory of perception. "As soon as this identification is rejected," we are told, "his whole system collapses: first his theories of space and time, then his belief in real contingency, then his condemnation of intellect, then his account of the relations of mind and matter, and last of all his whole view that the universe contains no things, but only actions, movements, changes, from nothing to nothing."

The theory that in pure intuition the subject which knows *becomes* its object, is certainly of very fundamental importance for Bergson. In justice, however, it must be remarked that this identification is not the result of a confusion; neither does it apply to thinking by concepts. Therefore Mr. Russell caricatures this theory when he says, "If subject and object are one....my friend Jones, though he believes himself to be in South America and to exist on his own account, is really in my head and exists in virtue of my thinking about him; St. Mark's Campanile, in spite of its great size

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[18] *Ibid.*, p. 23.

[19] *Ibid.*, pp. 25-26.

[20] *Ibid.*, p. 28.

[21] *Ibid.*, p .18.

[1] See *The Monist*, July, 1912, Vol. XXII, pp. 321-347.